Investigating the behavior of statistical indices for performance assessment of a reservoir

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SUMMARY

Commonly employed indices to assess the performance of a reservoir include reliability, resilience, and vulnerability. Depending upon the way in which resilience and vulnerability are computed, different values and behavior of these are obtained. This study has investigated the behavior of these indices using the data of an Indian reservoir. Synthetic data were generated for 1500 years employing long-memory and short-memory models. Monte Carlo simulation was carried out and the behavior of a number of performance measures was investigated. Keeping in view the sensitivity of measures and their monotonic behavior, time and volume reliabilities along with vulnerability were found to give suitable information about the performance of the reservoir. A reservoir sustainability index computed using time reliability and maximum vulnerability is proposed. Further, in semi-arid regions, evaporation losses are high and if these are ignored, one may overestimate the reliability of the reservoir by up to 10%.

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1. Introduction

Among the measures used in planning and management of reservoirs, time and volume reliabilities are employed most frequently. Time reliability indicates the proportion of time during an operating horizon for which the reservoir can meet the stipulated demands whereas volume reliability is the volume of water supplied as a fraction of the total target demand during the operating horizon. Hashimoto et al. (1982) proposed two more criteria – resilience and vulnerability – to measure the aspects that are not evaluated by reliability. Resilience is a metric which indicates how quickly a system recovers after a failure. A good system rapidly returns to a satisfactory state after a failure. If failures are prolonged and system recovery is slow vis-à-vis the behavior of the natural system, it implies that the system design is flawed. Obviously, a system with higher resilience would be preferred.

Vulnerability measures severity of failures, if and when they occur. If anticipated failure consequences of a low probability event are severe, strategies should be in place to deal with such eventualities. Here, the idea of safe fail as opposed to fail safe is important. If, due to a particular combination of inflows, demands, and storage capacity, some failures of a reservoir could be large, the reservoir would be highly vulnerable. Likewise, if a new industrial complex is expected to contaminate a nearby aquifer, one would state that vulnerability of ground water has increased.

While operating a storage reservoir, a failure is said to occur if demand during a particular month exceeds water supply. In the next month, the reservoir may remain in the failure state or may switch to success state. In the later case, the current failure is said to be over. A failure event has two aspects: the number of months in the event and the shortfall. Statistical indices highlighting different aspects of failure are described next.

1.1. Theoretical background

Failures in the operation of a reservoir have many aspects: number, extent, severity. In the following, the indices – reliability, resilience, and vulnerability (RRV) – that are used to measure different aspects of the performance of a reservoir are described. Usually these indices are computed using monthly or annual data but any other value of the time interval that is adopted for the operation of the system can be used.
1.2. Reliability

Two indices are generally followed in reservoir regulation. Time or occurrence based reliability is the probability that the system state lies in the set of satisfactory states

\[ r_t = 1 - (f_p/n); \quad 0 < r_t < 1, \quad f_p \leq n \]  

where \( r_t \) is the time reliability and \( f_p \) is the number of failure periods out of total \( n \) periods. Volume or quantity-based reliability \( r_v \) is expressed as

\[ r_v = V_s/V_d \]

where \( V_s \) is the volume of water supplied and \( V_d \) is the volume of water demanded during a given period.

Let \( N \) be the total number of failure events. Moving from time step \( t \) to \( (t + 1) \), the system will either remain in the same state or switch to the other state. The duration of the \( j \)th failure event is denoted by \( d_j \) and \( v_j \) is the corresponding deficit volume calculated by:

\[ v_j = \frac{d_j}{N} \left[ D_t - R_t \right] \]

where \( D_t \) is the target demand and \( R_t \) is the release from the reservoir for the month \( t \). Note that during the periods of high inflows, water may have to be spilled from the reservoir to ensure the safety of the dam. This extra outflow which does not serve any useful purpose is not considered to be release; the release used in Eq. (3) is always less than or equal to the demand \( (R_t \leq D_t) \). Therefore, the vulnerability will be zero in the periods when spill occurs.

1.3. Resilience

Resilience (\( \gamma \)) describes how quickly a system is likely to recover from failure. It is equivalent to the average probability of a recovery from failure in a single time step and may be equated to the inverse of the mean of the time that the system spends in an unsatisfactory state:

\[ \gamma_{\text{mean}} = \left[ \frac{1}{N} \sum_{j=1}^{N} d_j \right]^{-1} \]

Moy et al. (1986) defined resilience as the maximum consecutive duration of system remaining in unsatisfactory state. Kjeldsen and Rosbjerg (2004) defined resilience as the inverse of the maximum failure duration

\[ \gamma_{\text{max}} = \left[ \max_j \{d_j\} \right]^{-1} \]

According to Kundzewicz and Kindler (1995), the estimation of resilience based on maximum value is better than the mean value estimation because small insignificant events may lower the mean value. Kjeldsen and Rosbjerg (2004) compared these two estimates of resilience along with estimate using 0.9th fractile of the empirical Cumulative Distribution Function (CDF) of failure duration and deficit volume and advocated the use of long series of synthetic data to obtain robust estimates.

1.4. Vulnerability

Vulnerability measures the likely damage in a failure event and refers to the likely magnitude of a failure, if one occurs. Kjeldsen and Rosbjerg (2004) estimated vulnerability as the mean value of the deficit events \( v_j \) as:

\[ V_{\text{mean}} = \frac{1}{N} \sum_{j=1}^{N} v_j \]

Kundzewicz and Kindler (1995) suggested that the use of a maximum event might yield a better estimate of vulnerability:

\[ V_{\text{max}} = \max_j \{v_j\} \]

Generally \( V_{\text{max}} \) is in volumetric units, McMahon et al. (2006) used dimensionless vulnerability ratio by dividing \( V_{\text{max}} \) by target demand. Thus, there are different approaches to estimate resilience and vulnerability depending upon whether the mean or the maximum value of the variable denoting failure is adopted.

1.5. Sustainability index (SI)

In the recent past, some attempts have been made to quantitatively represent sustainability of reservoirs by using RRV. Zongxue et al. (1998) presented an index termed as drought risk index (DRI), composed of RRV:

\[ \text{DRI} = \beta_1 (1 - r_1) + \beta_2 (1 - \gamma) + \beta_3 (1 - V) \]

where \( \beta_1 + \beta_2 + \beta_3 = 1 \). No guidelines are available to select \( \beta \) weights. Loucks (1997) proposed a sustainability index (\( \kappa \)):

\[ \kappa = r_1 \gamma (1 - V_{\text{max}}/D) \]

where \( D \) is the draft.

1.6. Literature review

During the past 25 years, the above performance indices have been used in many studies such as Moy et al. (1986), Kundzewicz and Laski (1995), Vogel and Bolognese (1995), Kundzewicz and Kindler (1995), Vogel et al. (1999), and McMahon et al. (2006). To compute RRV indices, the required data includes the volume of water demanded and supplied for all periods, number of failure periods, and length and severity of failures. Monthly time period is most commonly used in published studies.

When many indicators of performance are available, a decision-maker may use some of them without fully knowing their characteristic behavior. Kjeldsen and Rosbjerg (2004) explored the monotonic behavior of RRV. They also addressing issues such as overlap and correlation between the estimators using synthetically generated data. Jain and Bhunya (2008) examined the behavior of RRV for a multipurpose reservoir with a probabilistic interpretation of their variation.

Vogel and Bolognese (1995) referred to earlier works where an index of reservoir performance known as resilience index or the standardized inflow or drift was defined

\[ m = \frac{(1 - x) \mu}{\sigma} = \frac{(1 - x)}{C_v} \]

where \( x \) is the annual yield of the reservoir expressed as a fraction of the mean annual inflow \( \mu \), \( \sigma \) is the standard deviation of the annual inflows, and \( C_v \) is the coefficient of variation of the annual inflows. Eq. (10) can also be written as:

\[ m = \frac{(1 - D/\mu)}{\sigma} = \frac{\mu - D}{\sigma} \]

Note that when \( \mu = D \), \( m = 0 \) and when \( \mu - D = \sigma \), \( m = 1 \). The following combinations of \( \mu, D \), and \( \sigma \) are important:

1. When \( (\mu - D) > \sigma \), \( m > 1 \). Such reservoirs are likely to have large fluctuations in the storage within a year and will be dominated by within-year behavior.
(2) When $\mu > D$ and $(\mu - D) \leq \sigma$, in these cases, $0 \leq m \leq 1$, and the reservoir will be dominated by over-year behavior. According to Vogel and Stedinger (1987), $m = 1$ is an arbitrary maximum for over-year behavior.

Vogel and Bolognese (1995) showed that $m$ is related to the probability that a storage reservoir will recover from a failure and hence it is a measure of reservoir resilience. Reservoirs with $m$ close to 0 require more time to recover from a failure than reservoirs with $m$ near unity. Systems with low resilience ($m$ near 0) are characterized by large values of $C_v$ or $\alpha$, or both. Reservoirs with $m$ near or above unity require less time to refill once empty.

Vogel et al. (1999) studied the behavior of reservoirs in the USA and found that over-year systems tend to have much lower resilience and slightly lower reliability than within-year systems. Over-year systems are much more vulnerable than within-year systems. They found that reservoir reliability and resilience are positively correlated.

McMahon et al. (2006) examined different reservoir performance metrics using data of four rivers and found that vulnerability ratio increases with streamflow variability. For a given draft, resilience was found to increase as reservoir size increases. It was observed that the resilience and vulnerability ratio is complementary. McMahon et al. (2006) also found that the variability of resilience and vulnerability decrease with increase in annual $C_v$ and suggested that the most effective way to reduce vulnerability for highly variable streamflows is by reducing the draft rather than increase the storage capacity.

Reservoirs with large $m$ tend to have either small demand levels, $\alpha$, or small $C_v$. Therefore for a fixed demand, one expects regions with low streamflow variability to contain more resilient reservoirs than regions with high streamflow variability. If demands from a reservoir increase with time, one would expect a reduction in the resilience. Besides, warmer climate will also result in higher irrigation and municipal demands and hence global warming is likely to reduce the resilience of the existing hydro-infrastructure.

The objective of this paper is to explore the behavior of performance indices of a reservoir such as RRV and duration of failure. Time series of synthetic data were generated and used. Since the focus reservoir is located in semi-arid region, an analysis was also conducted by ignoring and considering evaporation losses and the results have been compared.

2. Simulation experiments

Under consideration is a reservoir whose storage capacity is $S_{\text{max}}$. A long series of monthly inflows is available and the monthly demands are known. The reservoir is operated following the standard linear operating policy (SLOP), graphically represented in Fig. 1 (Jain and Singh 2003). Let $A_{\text{net}}$ represent the available water for the month $t$. Here $A_{\text{net}}$ is the sum of initial reservoir storage and inflow for the month $t$. Mathematically, the SLOP can be expressed as:

\[
\begin{align*}
\text{If } A_{\text{net}} &< D_t, & R_t &= A_{\text{net}} \\
\text{If } D_t &< A_{\text{net}} \leq S_{\text{max}} + D_t, & R_t &= D_t \\
\text{If } A_{\text{net}} &> S_{\text{max}} + D_t, & R_t &= A_{\text{net}} - S_{\text{max}}
\end{align*}
\]

(12)

The continuity equation for the reservoir is given by
\n
\[ S_t + I_t - E_t - R_t = S_{t+1} \]

(13)

where $I_t$ and $E_t$ represent inflow and evaporation loss for the reservoir for the month $t$; $S_t$ can vary from 0 to $S_{\text{max}}$. Evaporation loss is computed by using the average reservoir surface area and normal depth of evaporation for a period. Sometimes, observed actual depths of evaporation for a reservoir are not available for different months. Here ‘normal depth’ refers to the average depths of evaporation from the reservoir for different months, computed using the past observed data.

2.1. Application study

For this study, data pertaining to a reservoir located in a semi-arid region of India (the Dhari reservoir) were used. Mean annual rainfall in the catchment of this reservoir is 785 mm. Monsoon season (July–October) accounts for nearly 80% of the annual rainfall.

2.2. Data used

Monthly inflow series for the reservoir was available for 35 years. As the length of the series is inadequate to obtain stable estimates of performance indices, use of synthetic data was resorted to. Two concepts were employed to generate monthly flows: short-memory (SM) models and long-memory (LM) models. Short memory implies that the effect of a particular value of flow on the future values is negligible after a short period of time. The Thomas–Fiering model, which generates flows displaying such behavior, was employed to generate annual flows.

Long-memory implies that a particular value of the series influences future values for a long time. Many hydrological time series display long-term memory or persistence which is manifested by the tendency of wet or dry events to cluster together and this behavior is known as the Hurst phenomenon (Hurst, 1951). Fractional Gaussian noise (fGn) models reproduce the Hurst phenomenon; the symmetric moving average approach of Koutsoyiannis (2002) was used to generate annual flows.

Kjeldsen and Rosbjerg (2004) investigated about the length of record to give robust estimates of RRV and found that the estimates converged when a record length of 1000 years is used. Hence, in this study synthetic data series of 1500 years (18,000 months) were generated. Annual flows generated by SM and LM models were disaggregated into monthly flows by following the Method of Fragments (Srikanthan and McMahon 1980). In this method, the observed monthly flows are standardized year by year so that the sum of the monthly flows in any year equals unity. Thus, from a record of $n$ years, one will have $n$ fragments of 12 monthly flows. Now, the annual flows are ranked according to increasing magnitude, and $n$ classes were formed. The generated annual flows were then checked one by one for the class to which they belong and were disaggregated using the corresponding fragment.

For the historical data, mean annual runoff (MAR) was 858.193 MCM and its standard deviation (SD) was 694.791 MCM. In case of LM model, the mean and SD of generated annual series were 858.193 MCM and 583.597 MCM and for the SM model, these statistics were 858.193 MCM and 733.683 MCM, respectively. For the
annual inflow data, the correlation coefficient was \(-0.167\) and the value of Hurst coefficient was 0.708. For the generated data, correlation and the Hurst coefficients were \(-0.052\) and 0.656 respectively. Data of monthly demands for all uses as provided by the dam authorities were used in the study. The storage capacity of the reservoir used was 829 million cubic meter. The annual demands from the reservoir are 46.3% of MAR. Employing the criteria of Vogel et al. (1999), this reservoir is over-year storage. In the numerical experiments, the draft ratios (draft/MAR) used were: 0.3, 0.35, 0.4, 0.45, \ldots, 1.0. Further, storage capacities corresponding to four storage ratios (Smax/MAR) were used: 0.5, 0.7, 0.9, and 1.1.

3. Results and discussion

Using the two sets of generated inflows, the operation of the Dharoi reservoir was simulated following SLOP for two cases: (a) by ignoring evaporation losses and (b) by considering evaporation losses. For each case, eight performance indices were computed: time and volume reliability, mean and maximum resilience, mean and maximum vulnerability, number of failure events, and total number of failure months. In the following, the simulation results are discussed in detail.

3.1. Consideration of evaporation

In some reservoir operation planning studies, evaporation losses are ignored. However, for reservoirs located in semi-arid or arid areas, the loss of water due to evaporation could be quite high and if ignored, one does not get realistic values of performance indices. For the Dharoi reservoir, the normal monthly evaporation depths were available. January had the least evaporation (0.1402 m) while May accounted for the highest (0.3048 m) depth of evaporation; the annual depth of evaporation was 2.15 m. In the simulation studies, consideration of evaporation loss had different impact on various indices.

Fig. 2a and b present the number of failure months for the situations when evaporation was considered and ignored, respectively. Expectedly, for all cases the number of failure months increased when evaporation is accounted for. As the storage ratio increases, the number of failure months reduces significantly. For instance, for LM model inflows, draft ratio 0.6, and storage ratio 0.5, the time reliability was 0.893 when evaporation losses were ignored and was 0.862 when these were considered. Further, time and volume reliability and mean vulnerability increased and the number of failures decreased when evaporation is ignored; maximum vulnerability and maximum resilience were insensitive to it. Mean resilience was found to be quite sensitive to inclusion of evaporation. The curves of mean resilience vs. draft ratio were not monotonic and their shapes were different in the two cases. The pattern of behavior was the same for the inflows generated by LM and SM models.

3.2. Failure events

A reservoir may fail to meet the demands due to several reasons such as inadequate inflows, high magnitude of demand, loss of water from the reservoir due to seepage, evaporation, etc. Here, attempt was made to estimate the difference in failure events in a simulation study considering evaporation loss and neglecting it. Expectedly, in simulation runs, the number of failure events and the total failure duration was found to depend whether evaporation is considered or ignored. For example, for storage ratio 1.1 and draft ratio 0.7, there were 86 failure events and failure duration was 486 months when evaporation losses were considered but these reduced to 13 and 90 respectively when evaporation losses were ignored. Further, for the storage ratio of 0.7 and draft ratio of 0.5, failure events were 77 and failure duration was 451

![Fig. 2. Variation of number of failure months with draft ratio: (a) LM flow model – ignoring evaporation losses, (b) LM flow model – considering evaporation losses, (c) SM flow model – ignoring evaporation losses, and (d) SM flow model – considering evaporation losses.](image-url)
when evaporation losses were considered but these reduced to 21 and 145 respectively when evaporation losses were ignored. These results again highlight the importance of evaporation losses in semi(arid) climates.

3.3. Reliability

Fig. 3 shows volume reliability as a function of draft/MAR for different values of storage ratios for SM flows and considering evaporation. Volume reliability drops monotonically as draft/MAR increases and higher values are obtained as $S_{\text{max}}$/MAR increases. Similar behavior was observed for time reliability also. It may be noted that the gap between the curve for storage ratios 0.5 and 0.7 is more compared to that between 0.7 and 0.9, and so on. This implies that as larger storage is provided, improvements in reliability are progressively smaller. Note that the slope of the curve for storage ratio 0.5 changes around draft ratio 0.6. At this value, standard inflow $m = c_v$ which is the threshold to classify the reservoirs as within-year or over-year storage.

Both time and volume reliabilities are frequently used to evaluate the performance of a system because these are easy to compute, are intuitively appealing, and change monotonically.

3.4. Resilience

Graphs of mean and maximum resilience as a function of draft/MAR for different values of $S_{\text{max}}$/MAR are shown in Figs. 4a and 4b. Among the measures of resilience, the mean resilience was found to decrease non-monotonically when draft/MAR ratio increases and it was found to be insensitive to storage ratio at large values of draft ratios. Due to this, mean resilience does not qualify to be a suitable performance metric. Further, the maximum resilience showed a monotonically decreasing trend. At high as well as low values of draft/MAR ratio, maximum resilience was insensitive to $S_{\text{max}}$/MAR as well as draft/MAR ratio for SM models. Fig. 4b shows that the reservoir is highly resilient when the demands are low and is not resilient when the demands are high.

3.5. Vulnerability

Fig. 5 shows the variation of mean vulnerability as a function of draft ratios for different values of storage ratios for data generated by the LM model. Mean vulnerability curves for different storage ratios fall in a narrow band and vulnerability increases non-monotonically with draft ratio. Kjeldsen and Rosbjerg (2004) also noted that mean vulnerability changes non-monotonically with draft ratio and in this study also, the same behavior of these indices was noticed. Further, the maximum vulnerability (Fig. 6a and b) was found to show a monotonic rise with the draft ratio for flows of both memory models. It is also noted that the maximum vulnerability is quite sensitive to storage ratio for SM model. Hence, maximum vulnerability is a suitable indicator of reservoir performance.

3.6. Long- and short-memory models of inflows

Simulation results indicated that, between the long- and short-memory models, in general, the difference in performance indices by considering and ignoring evaporation losses were more for LM models. Also, for the same draft ratio, the difference in indices for two different storage ratios was larger for flows pertaining to LM model. For example, for $S_{\text{max}}$/MAR of 0.5 and 1.1 and draft ratio 0.9, the difference in volume reliabilities using LM model data was 0.082 while the same using SM model data was 0.065; reliabilities were higher using data of SM model. While examining the use of long-memory flow models in reservoir analysis, Klemes et al.
(1981) had remarked that for a long time to come, the use of long-memory models will, in principle, remain equivalent to the use of a small safety factor in the intrinsically inaccurate estimate of reservoir performance reliability.

3.7. Sustainability index

In absence of an objective method to select weights of DRI Eq. (8), sustainability index \( \kappa \) given by Eq. (9) was computed. When SI was plotted against draft ratio (see Fig. 7), its variation was non-monotonous which means that this is not a suitable index. Results of earlier studies which were confirmed in this work show that resilience and vulnerability are correlated. Therefore, an alternative index of sustainability (IS) is proposed as follows:

\[
K_{\text{new}} = r_f(1 - V_{\text{max}}/D)S_{\text{max}}/\text{MAR}
\]  

A higher value of IS will imply that the reservoir is more sustainable. Here, as the time reliability increases, \( K_{\text{new}} \) also increases. Further, as the maximum vulnerability increases, the value of \( K_{\text{new}} \) decreases, implying that the reservoir becomes less sustainable which is intuitively correct. Finally, IS is scaled by \( S_{\text{max}}/\text{MAR} \) which makes the index sensitive to this ratio. Variation of \( K_{\text{new}} \) with draft ratio is plotted in Fig. 8 which shows that \( K_{\text{new}} \) has a monotonous variation with draft ratio. Thus, Eq. (14) presents a suitable sustainability index for storage reservoirs.

4. Conclusions

Behavior of several performance indicators for a storage reservoir has been investigated. Based on the results, it is recommended that evaporation losses from the reservoirs be considered even in planning stages, particularly if the reservoir is located in (semi)arid regions. Ignoring evaporation, one may significantly overestimate reservoir reliability. Results obtained in this study are in line with the observations by earlier studies that the estimates of resilience and vulnerability based on mean values of failure duration and deficit volume are non-monotonic. This study also showed that maximum resilience and vulnerability change monotonically but these are not very sensitive to changes in storage ratios at high and low draft ratios. Hence, considering monotonic behavior, sensitivity, and estimation uncertainty, time and volume reliability and maximum vulnerability are suitable indicators of reservoir performance. An index of sustainability based on time reliability, vulnerability and storage to mean annual runoff ratio \([\text{Eq. (14)}]\) is proposed as a composite indicator of reservoir performance.

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